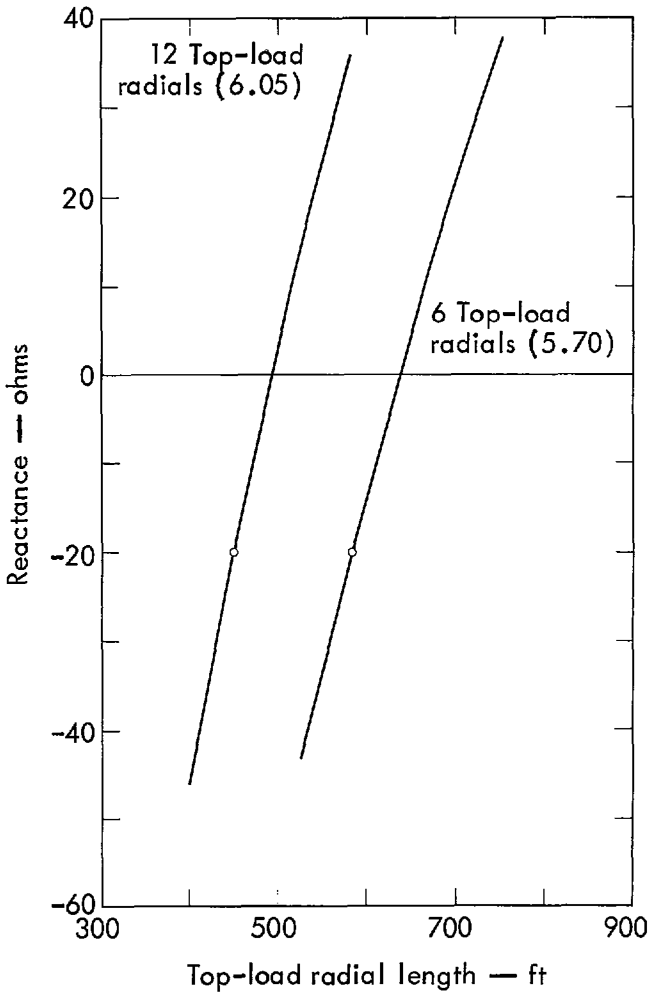

Fig. 5. Reactance of 1200-ft top-loaded monopole at 100 kHz as function of load radial length. $B\eta$ product in kHz is shown in parentheses for $-j20$ load length.

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A Wide-Band Square-Waveguide Array Polarizer

MING HUI CHEN AND G. N. TSANDOULAS

***Abstract*—A stepped-septum polarizer has been designed that is capable of at least 26 dB of isolation over a 20 percent frequency band. The three-port device may be used to excite high purity left or right circular, as well as tilted linear polarizations in a phased array of square waveguides.**

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Introduction

Phased array technology has been increasingly concerned with multiply polarized arrays [1], [2]. These are arrays capable of supporting more than one sense of polarization and which depend for their operation on symmetric radiators such as circular or square waveguides and crossed dipoles. This paper describes a simple and effective three-port device that can be used to feed square-waveguide radiators in such a way as to excite high purity left-hand or right-hand circular polarizations. The input consists of two identical rectangular waveguide ports while the output is a square waveguide. By introducing additional hardware and exciting both input ports simultaneously with the proper phase relationship, the device can also be used to produce linear polarization that is tilted at any angle with respect to the horizontal or vertical aperture direction. A two-port circular polarizer, without tilted-linear polarization capability, using irises in square waveguide was described some time ago [3].

The basic H-plane sloping-septum polarizer was studied experimentally several years ago [4]. In this communication a modified version employing a stepped-septum is presented. Since this modification incorporates more design parameters, such as the step lengths and widths of the septum, it is expected that better performance in terms of axial ratio and input-port isolation will result. That this is indeed the case has been confirmed experimentally and the results are presented in the following section.

Phenomenological Analysis of Stepped-Septum Polarizer

A stepped-septum polarizer is schematically represented in Fig. 1(a). For purposes of analysis we consider it as a four-port network as in Fig. 1(b). Although this network will not be solved completely due to the difficulty in modeling the wave coupling mechanism in the septum region, the symmetry properties of the structure will be used to set up relationships among the network parameters which provide guidelines for designing the device. The following phenomenology assumes that all higher-order modes are cut off and that the septum is infinitely thin.

A. Even-Mode Excitation

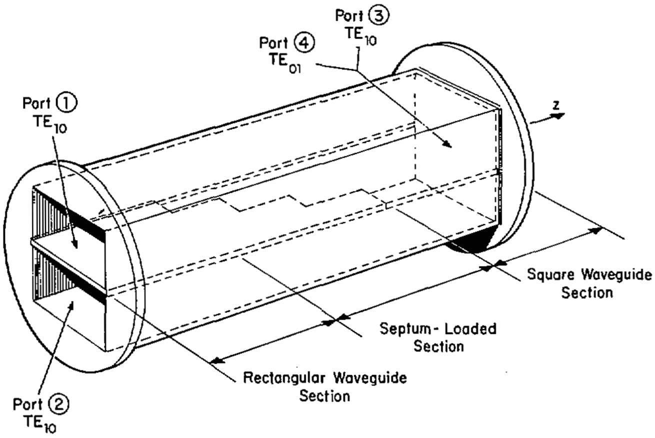
A traveling-wave description of this structure defines a_i and b_i as the incident and reflected wave amplitudes, respectively, at port i ($i = 1,2,3,4$). When an even mode is excited in the rectangular waveguide region (i.e., $a_1 = a_2 = 1$, $a_3 = a_4 = 0$), the electric- and magnetic-field distributions in the upper and lower rectangular waveguide are the same but the currents on each side of the common wall flow in opposite directions. Therefore, a slot in the common wall between these two guides will not disturb the field and the current flow in one guide will simply be the return current of the other guide through the slot. Consequently, the propagation of the even-mode wave will be unaffected by the septum and will transfer its total energy into the TE_{10} mode in the square waveguide.

B. Odd-Mode Excitation

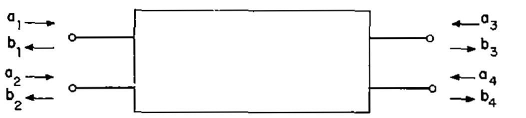
When an odd mode is excited in the rectangular waveguide region (i.e., $a_1 = 1$, $a_2 = -1$, $a_3 = a_4 = 0$), the fields and current flow in the lower waveguide are reversed compared to the even-mode case. This causes the current flows on the top and bottom surfaces of the common wall between the two waveguides to be in the same direction and a slot in this common wall will cause a field disturbance, resulting in mode coupling and reflection.

The transverse field of an odd mode in a dual rectangular waveguide is characterized by odd up and down symmetry whereas the TE_{10} mode in the square waveguide possesses an even up and down symmetry, which leads to no coupling between them. Therefore, the odd-mode energy will be partially transferred to the TE_{01} mode in the square waveguide and partially reflected.

The excitation of port 1 is equivalent to the superposition of even- and odd-mode excitations. This is the case when a circular



(a)



(b)

Fig. 1. (a) Polarizer-square waveguide configuration. (b) Polarizer equivalent network.

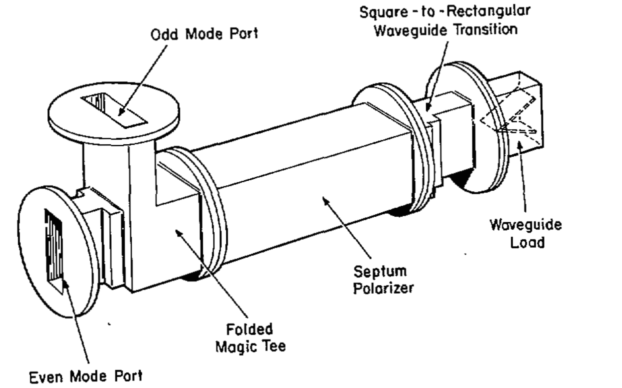


Fig. 2. Experimental setup for even- or odd-mode reflection coefficient measurement.

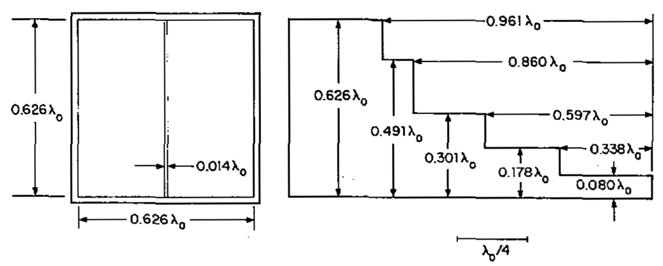


Fig. 3. Prototype stepped-septum polarizer to scale. λ_0 is free-space wavelength measured at midband frequency. Septum thickness is not critical.

polarizer is in operation. For this case we have

$$b_1 = \frac{\Gamma}{2} = -b_2$$

$$b_3 = \frac{1}{2}$$

$$b_4 = \frac{1}{2}\,(1 - |\,\Gamma\,|^2)^{1/2} e^{j\theta}$$

where Γ is the reflection coefficient for odd-mode excitation, b_1 represents the power reflected back to the input port, which determines the VSWR of the device, and b_2 represents the power coupled to the nonexcited input, which determines the isolation. The relative amplitude and phase of b_3 and b_4 determine the axial ratio and major-axis tilt angle of the elliptically-polarized wave. For a perfect circularly-polarized wave, b_3 and b_4 must be equal in amplitude and 90° out of phase. Therefore, the design objective is to minimize $|\,\Gamma\,|$ and keep the output phase angle as close to 90° as possible.

Prototype Stepped-Septum Polarizer

Complete field solutions for the septum region of a polarizer (Fig. 1) are very difficult to arrive at and will not be attempted here. In addition, analytical design methods similar to those for impedance transformers or directional couplers are not available at the present time. Based on the symmetry properties outlined previously and the interrelationship between device performance and equivalent network parameters, we have arrived at a successful design by trial and error experimental methods and have subsequently generalized the results.

Minimizing Odd-Mode Reflection Coefficient

Fig. 2 shows a setup suitable for providing even- or odd-mode excitation to the polarizer. The $E(H)$ plane port of a folded magic tee corresponds to the input port for even-(odd)-mode excitation. The polarizer partially couples some of the even-(odd-) mode power to the TE_{10} (TE_{01}) mode in a square waveguide, which is terminated in a load by a square-to-rectangular-waveguide taper,[1] and partially reflects the uncoupled energy back to the input port. It is assumed that, because of orthogonal symmetry, there is no coupling between the TE_{10} and TE_{01} modes. In the present device the measured cross coupling between these modes was less than −40 dB. A VSWR swept frequency measurement at the H-plane port of the magic tee yields the odd-mode reflection coefficient $|\,\Gamma\,|$. Then the main effort is directed toward repeating the process of odd-mode reflection coefficient measurement by changing the step widths and lengths until a performance goal is obtained. A designed prototype polarizer using a four-step septum is shown in Fig. 3.

The VSWR is less than 1.2 over a 20 percent frequency band which is equivalent to 26 dB of isolation. (A minimum of 20 dB of isolation has been obtained for a sloping-septum polarizer over a 10 percent frequency band [5]). Transmission loss measurements indicate that the structure has ohmic losses of less than 0.1 dB and that the amplitude difference between the TE_{10} and TE_{01} modes is about 0.2 dB. A near unity axial ratio may therefore be expected if the 90° phase orthogonality can be met.

Obtaining Phase Orthogonality Between TE_{10} and TE_{01} Modes

An adequately designed H-plane stepped-septum excites the TE_{10} and TE_{01} modes in approximately equal amplitudes. However, a phase adjustment between them was found to be necessary for good phase orthogonality. This was a consequence of our design philosophy under which the four-step septum was configured to give minimum reflection (maximum isolation). This quantity was found to be more critically dependent on septum geometry than phase. Quite probably, a modification of the septum dimensions may yield an adequate phase relationship, at the expense of some isolation, thus obviating the need for subsequent phase correction.

One way to achieve good phase orthogonality is by inserting a thin dielectric slab into the square waveguide following the septum (Fig. 4). The slab delays one mode of propagation relative to the other. The slab dimensions for this polarizer were obtained using an experimental rotating horn set-up for measuring axial ratio. With the Teflon slab ($\epsilon_r' = 2.1$) of Fig. 4 in place, the final per-

[1] A 90° rotation in the square-to-rectangular-waveguide taper is necessary to switch the mode of operation from TE_{10} to TE_{01}.

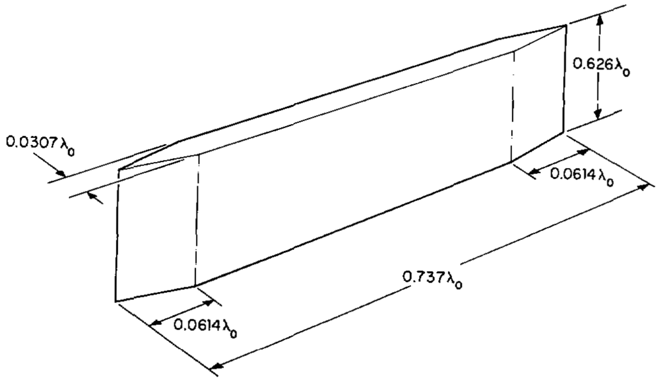


Fig. 4. Geometry of Teflon slab loading for optimum axial ratio condition. Dimensions are not critical.

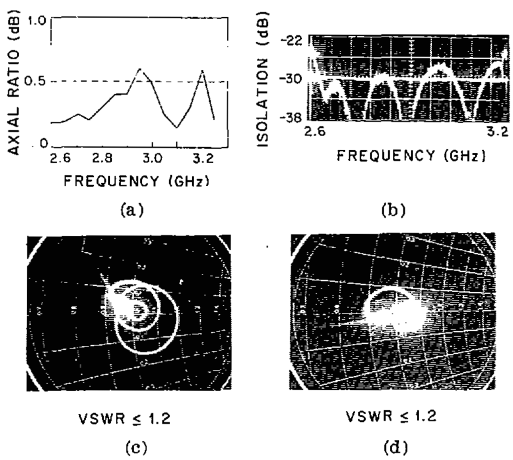


Fig. 5. Performance of stepped-septum polarizer with Teflon slab of Fig. 4 in place. (a) Axial ratio with port 1 excited and port 2 match loaded. Port 2 performance was similar. (b) Isolation. (c) VSWR of port 1—right-hand circular polarization. (d) VSWR of port 2—left-hand circular polarization. VSWR is less than 1.2 for both (c) and (d) which are expanded 1.5 VSWR Smith chart plots.

formance of the polarizer is shown in Fig. 5. It is seen that while improving the axial ratio the slab did not adversely affect either the VSWR or the isolation.

Comments

The stepped-septum polarizer described in this communication gives significantly better performance over a wider bandwidth than the sloping-septum device.

In an actual phased-array application, because of lattice limitations, the square-waveguide radiators will almost certainly have to be dielectrically loaded for scan ranges of the order of $\frac{1}{4}$ hemisphere or greater [2]. In such cases all dimensions of the polarizer of Fig. 3 are to be scaled by the factor $1/(\epsilon_r)^{1/2}$ where ϵ_r is the dielectric constant of the material filling the waveguide. The same scaling applies to the dimensions of the phase-correcting dielectric slab (Fig. 4) which will have to be of dielectric constant ϵ_r' such that $\epsilon_r' = 2.1\epsilon_r$. This method of correcting phase may not be suitable if the device is used in high-power applications and is, in general, very cumbersome for dielectrically filled waveguides. A simpler method for both cases would be to introduce a small step in two opposite walls of the square waveguide following the septum in order to delay one mode relative to the other.

At first sight it might appear that the size of the square waveguide is too restrictive ($a = 0.626\ \lambda_0/(\epsilon_r)^{1/2}$). This is a consequence of the fact that square waveguides are capable of only about 34 percent bandwidth, under the already stated assumption that all higher order modes are below cutoff. A 20 percent bandwidth capability thus leaves a margin of about 14 percent, part of which ($\sim$12 percent) is used as guard between the lower edge of the operating band and the waveguide dominant-mode cutoff, for good matching, while the remainder ($\sim$2 percent) is the guard between the upper edge of the operating band and the cutoff of the first (TE_{11}) higher order mode. These margins may be juggled somewhat but probably not appreciably so that a restriction on waveguide size may be met by choosing an appropriate dielectric material. It is believed that a 25 percent bandwidth is close to the maximum practical operating bandwidth for a square-waveguide polarizer or radiator [2] free of higher order modes.

Although this polarizer may be used for any bandwidth less than 20 percent as it stands, a reoptimization procedure would probably yield better performance. For narrow bandwidths ($\sim$2–5 percent) the sloping-septum polarizer [4] appears to be adequate.

Acknowledgment

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On the Scattering Cross Section of Passive Linear Arrays

L. SOLYMAR

***Abstract*—A general formula is derived for the scattering cross section of a passive n-element linear array consisting of isotropic radiators. When all the reactances are tuned out and scattering in the mirror direction is investigated, it is found that A_{sr}, the relative scattering cross section is equal to the square of the maximum gain the array can produce. As a consequence, for forward scattering in the limiting case of zero spacing between the elements, $A_{sr} = n^4$.**

The cross section of a scattering object having cylindrical symmetry is defined as follows

$$A_s(\theta_1,\theta_2) = 4\pi r^2 \frac{\text{power density at distant point } r \text{ in direction } \theta_2}{\text{power density of plane wave incident from direction } \theta_1}. \quad (1)$$

For an array one is more interested in the relative scattering cross section

$$A_{sr}(\theta_1,\theta_2) = \frac{A_s(\theta_1,\theta_2)}{A_{si}(\theta_1,\theta_2)} \quad (2)$$

where A_{si} is the scattering cross section of the individual radiator.

In this Communication we shall express $A_{sr}(\theta_1,\theta_2)$ in terms of the geometry of the array and of the parameters of the matching matrix. A general formulation for an arbitrary three-dimensional array is certainly possible, but we shall, for simplicity, restrict the investigation to linear arrays consisting of isotropic radiators. In that particular case there are several theorems available making

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